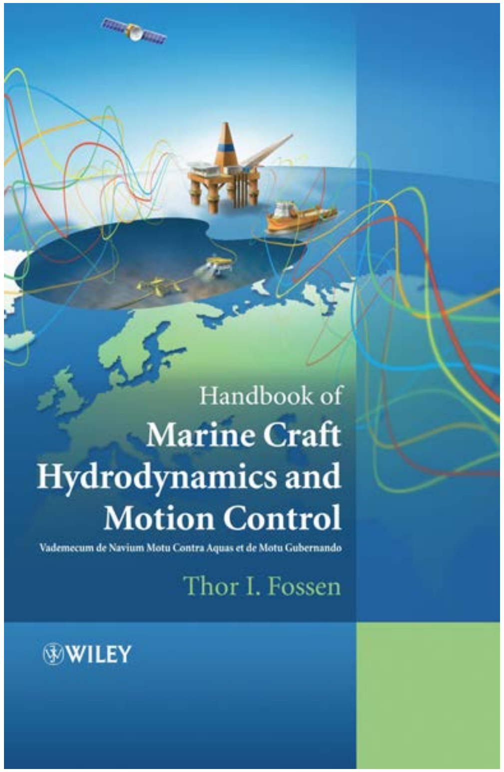
Lecture Notes:
TTK 4190 Guidance and Control of Vehicles

Professor Thor I. Fossen
Department of Engineering Cybernetics
NTNU Centre for Autonomous Marine Operations and Systems
Norwegian University of Science and Technology (NTNU)

Home page: http://www.fossen.biz/

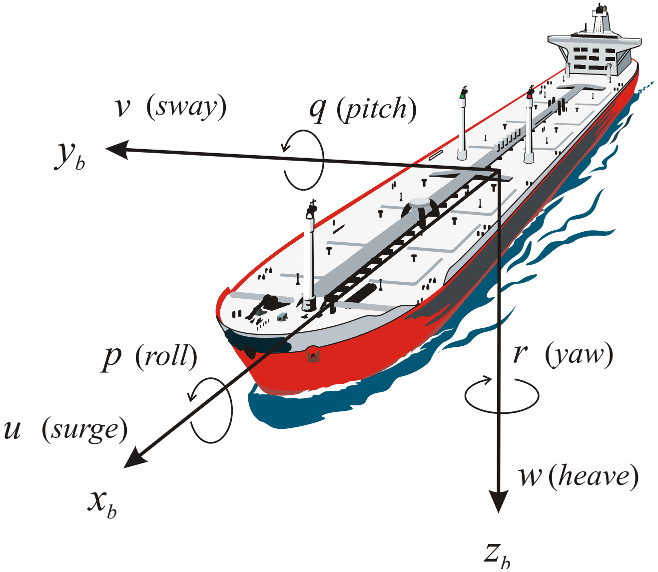
Textbook and Notation



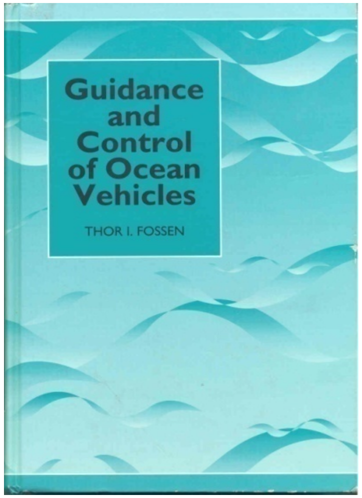
Fossen, T. I. (2011)
Handbook of Marine Craft Hydrodynamics and Motion Control. John Wiley & Sons Ltd.

MSS Toolbox: www.marinecontrol.org

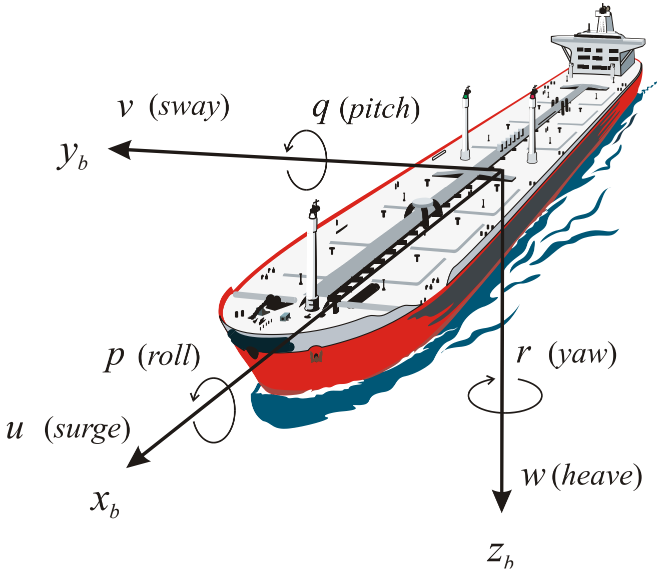
✓ **SNAME (1950).** Nomenclature for Treating the Motion of a Submerged Body Through a Fluid. *The Society of Naval Architects and Marine Engineers, Technical and Research Bulletin No. 1-5, April 1950, pp. 1-15.*



Useful Reference Book

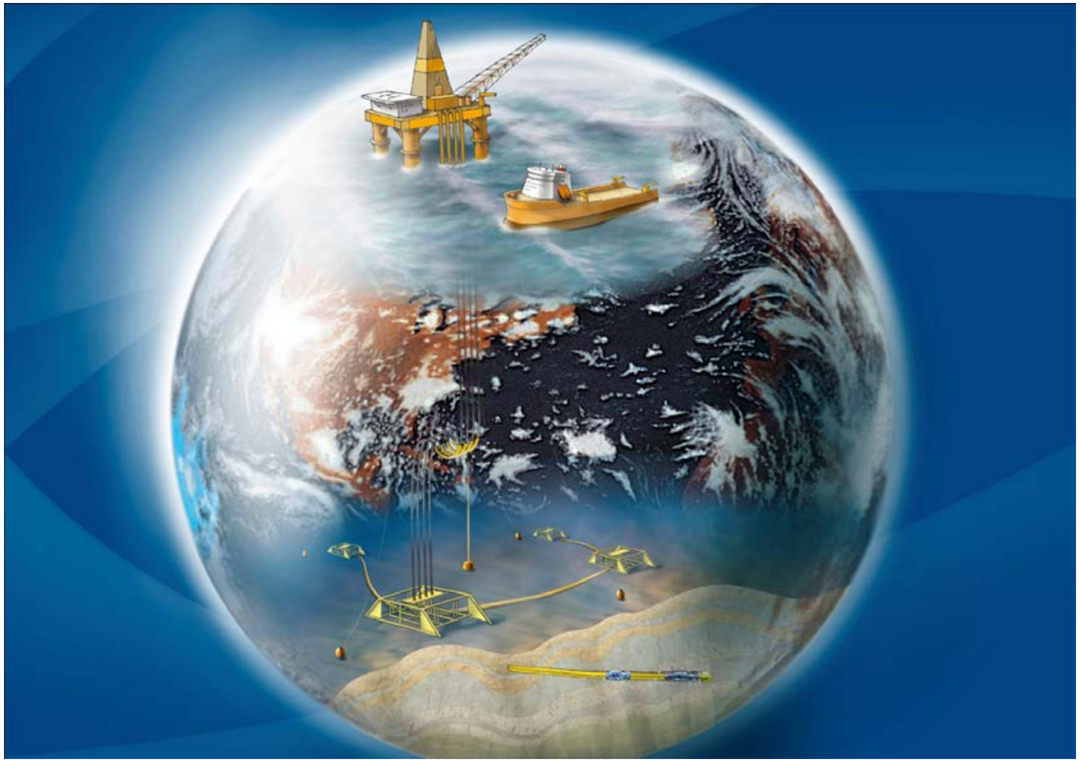


- ✓ **Fossen, T. I. (1994).** *"Guidance and Control of Ocean Vehicles", John Wiley & Sons Ltd.* ISBN 0-471-94113-1



Goals for the Course:

1. Mathematical modeling of vehicles. This includes:
   - Kinematics
   - Kinetics
   - Equations of motion for marine craft and aircraft
   - Wind, wave and ocean current models
   - Hydrodynamics: maneuvering and seakeeping theory



2. Design of guidance, navigation and motion control systems for a large number of applications
3. Simulate the motions of marine craft and aircraft in the time-domain using hydrodynamic/aerodynamic models

Marine Craft in Operation

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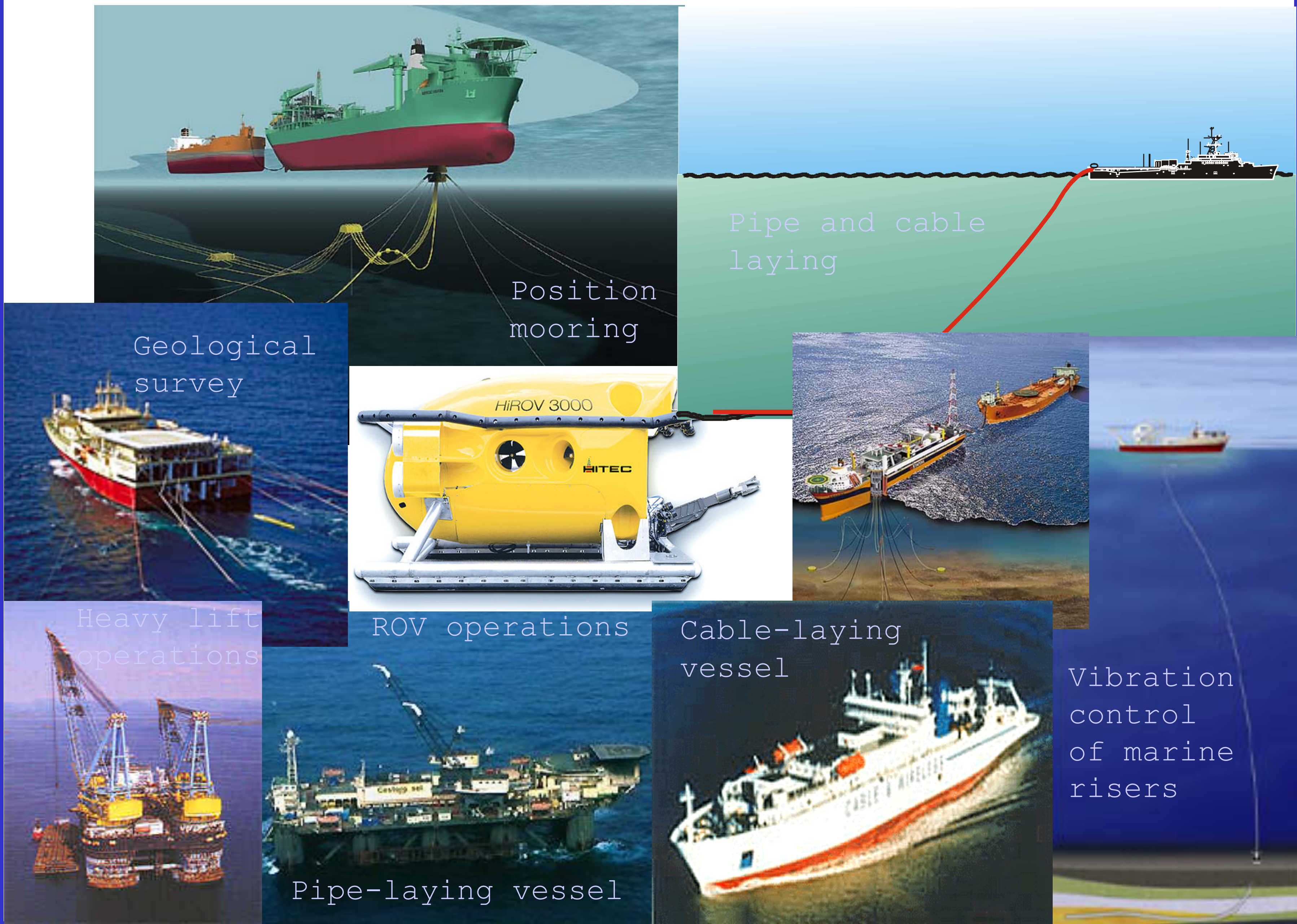
Marine Craft in Operation



NTNU

Path Following and Trajectory Tracking

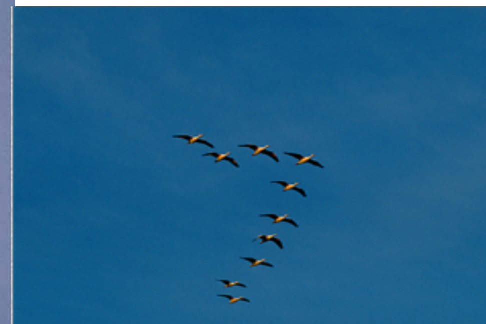
Fully actuated supply ship cruising at low speed.

Underactuated container ship in transit.

Italian supply ship **Vesuvio** refueling two ships at sea.

Courtesy: Hepburn Eng. Inc.

Formation Control/Underway Replenishment



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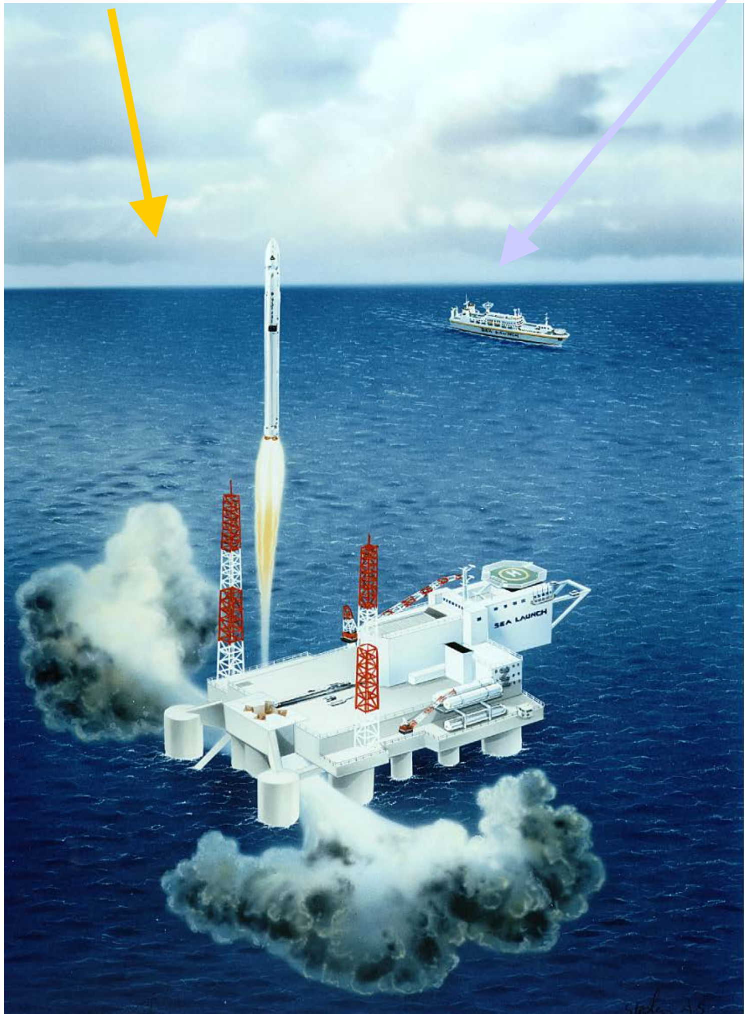
Interdisciplinary: Rocket Launch / DP system / THCS

Launch Platform

Assembly Command Ship



Courtesy: SeaLaunch LLC

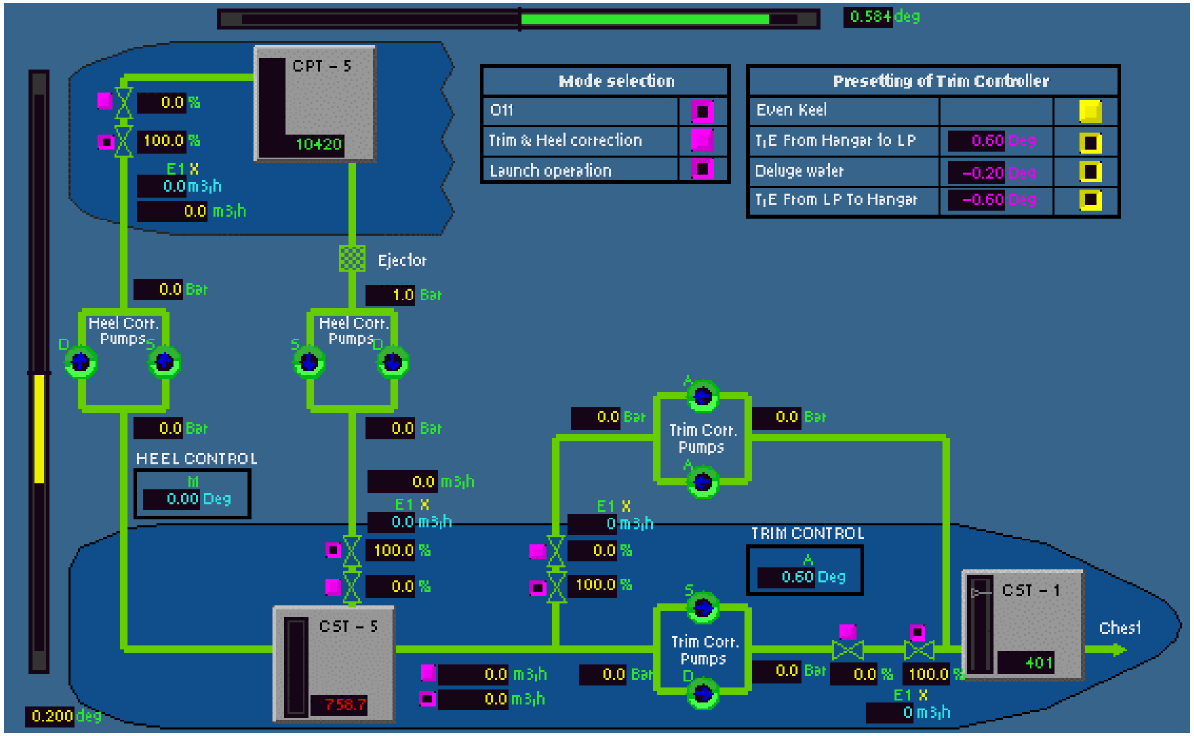
SEA LAUNCH SM

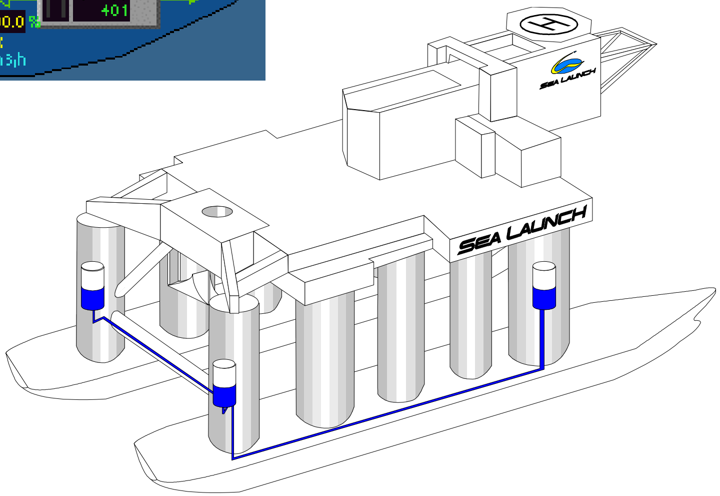
Marine Segment

Courtesy: SeaLaunch

http://www.sea-launch.com

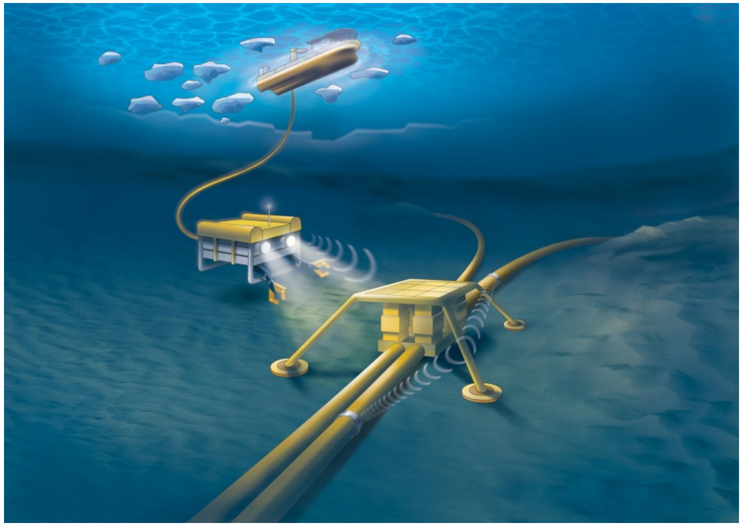
Trim & Heel Correction System (THCS)





Process and Marine Control

NTNU Infrastructure



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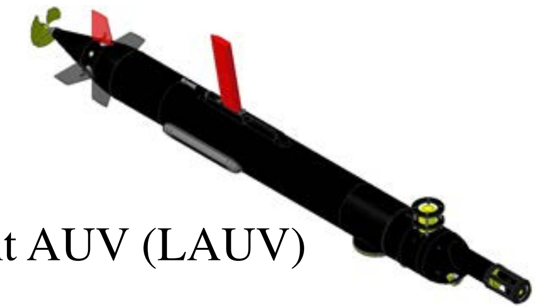
Applied Underwater Robotics Laboratory

The fleet of the AUR-Lab at NTNU Trondheim

https://www.ntnu.no/aur-lab

Remus 100 AUV



Light AUV (LAUV)

Hugin AUV

ROV Minerva

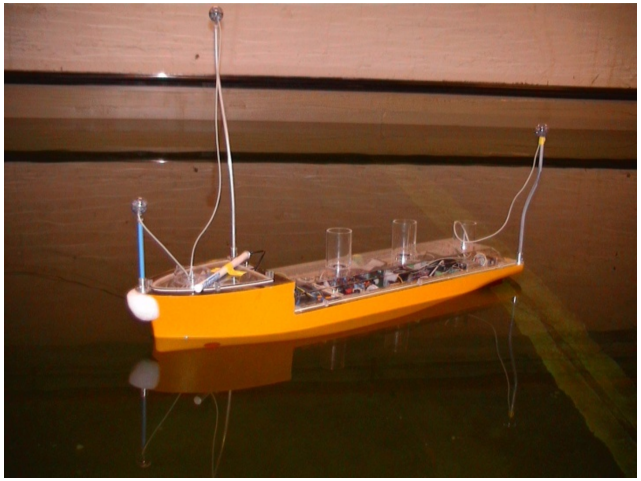
Marine Cybernetics Laboratory (MCLab)

MCLab Dimensions:

$$L \times B \times D = 40\ \mathrm{m} \times 6.5\ \mathrm{m} \times 1.5\ \mathrm{m}$$

✓The software is developed by using rapid prototyping techniques and automatic code generation under Matlab/Simulink™ and RT-Lab™.

✓The target PC onboard the model scale vessels runs the QNX™ real-time operating system, while experimental results are presented in real-time on a host PC using Labview™.



Cybership II

NTNU Research Vessel Gunnerus

31 meters long

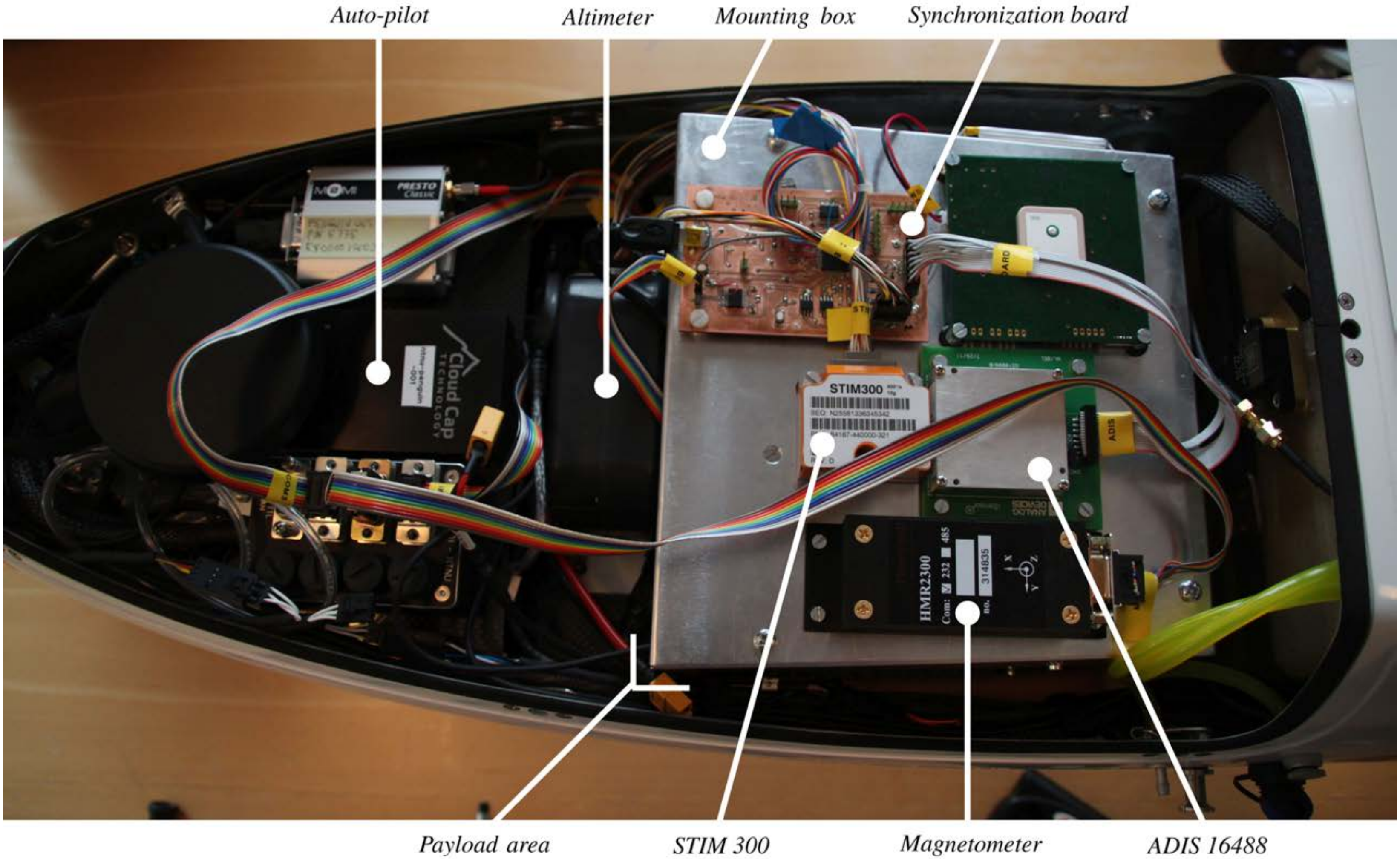
Top speed 13 knots

http://www.ntnu.edu/marine/gunnerus

UAV Factory Penguin B w/ Piccolo SL

- 28 m/s cruise speed
- Gasoline, 8 hr endurance
- MTOW 21 kg
- 2-5 kg payload capacity
- Large payload bay
- 80W generator
- Avionics system integration made with Maritime Robotics based on Cloudcap technology
- Telemetry on 2.4 GHz radio, GPRS (and VHF)
- Catapult launch
- Custom payload system integration with avionics interface

Penguin equipped with Camera, INS and GPS Sensor Suite for Data Logging



Penguin navigation payload: Two IMUs, optical camera, infrared camera, RTK GPS and embedded controller for data logging

Skywalker X8 w/Ardupilot

- 18 m/s cruise speed
- Catapult launch
- Belly or net landing
- Electric, 1hr endurance
- Large payload bay
- >1 kg payload capacity
- Inexpensive
- Flexible avionics and payload system integration with ArduPilot open source autopilot and mission planning SW
- Currently telemetry on 433 MHz or 5.8 GHz radio for VLOS
- Can be set up for BLOS with GPRS and VHF radio links

3DRobotics hexa-copter w/Ardupilot

- Electric, 5-10 min endurance
- 1 kg payload capacity
- Inexpensive
- Flexible system integration with Ardupilot open source autopilot and mission planning SW
- Telemetry on 433 MHz og 5.8 GHz radio

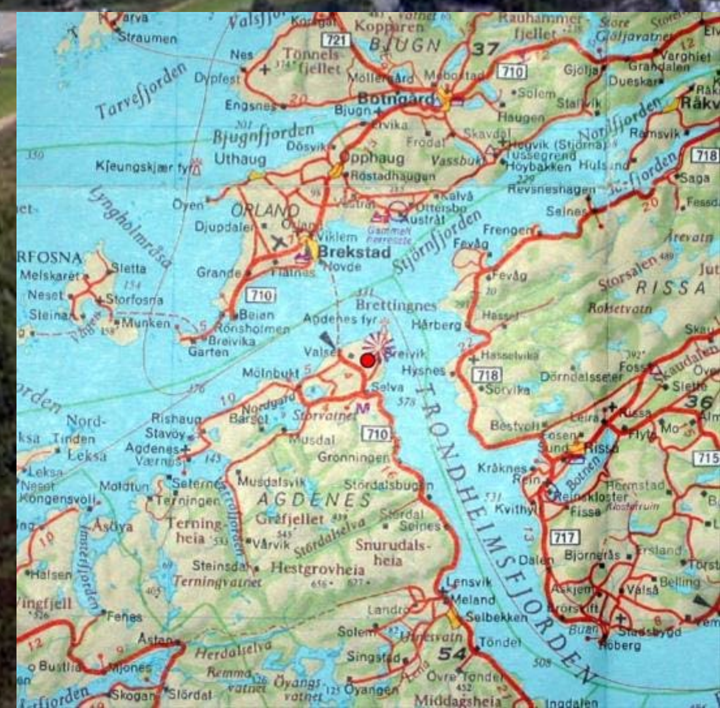
Microdrone Quad-copter

- Turn-key solution
- Various camera, video and radio systems
- Electric, 45 min endurance
- 2-3 kg payload capacity

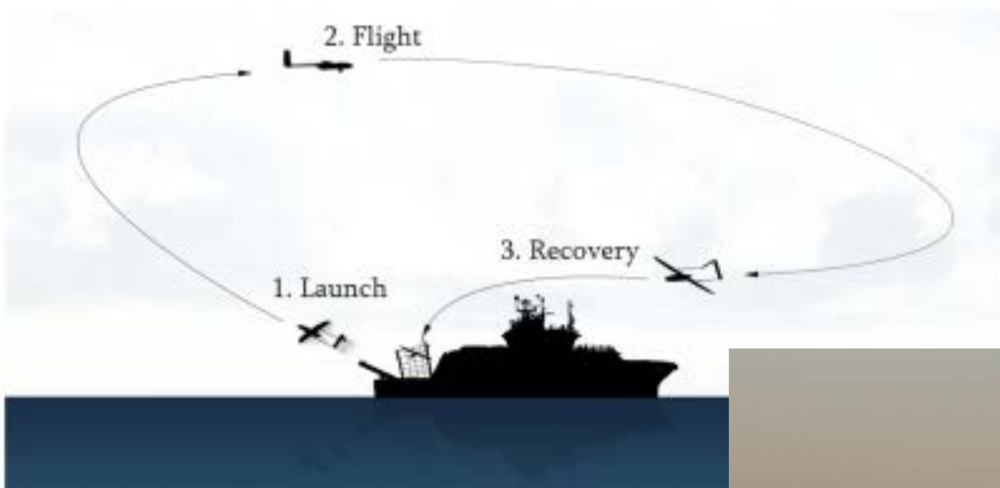
NTNU Airfield at Agdenes

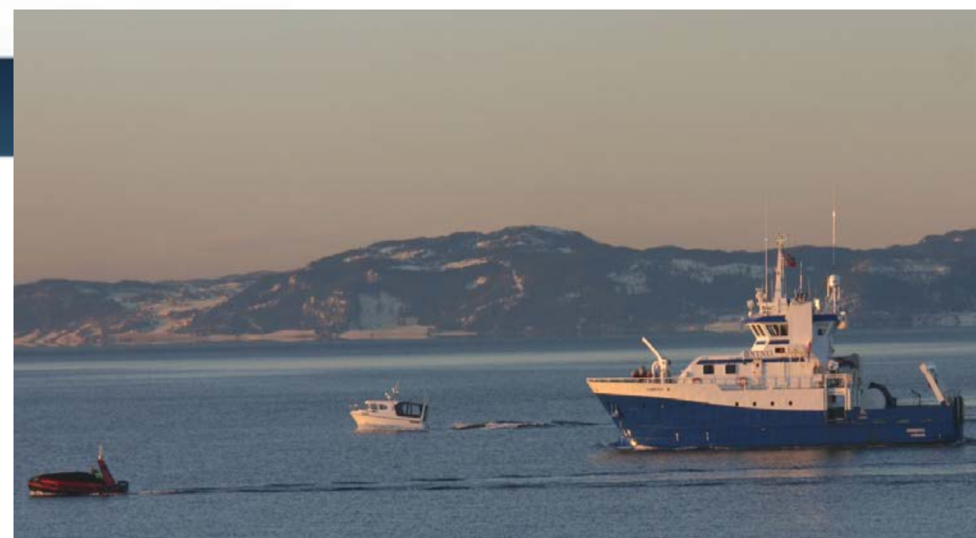
Located 94 km North-West of Trondheim

We also use the airports at Eggemoen and Ørland



Offshore UAV Launch and Recovery Systems





Launching – The Easy Part
“Human-Inspired Approach”

Automatic Launch of the Penguin UAV

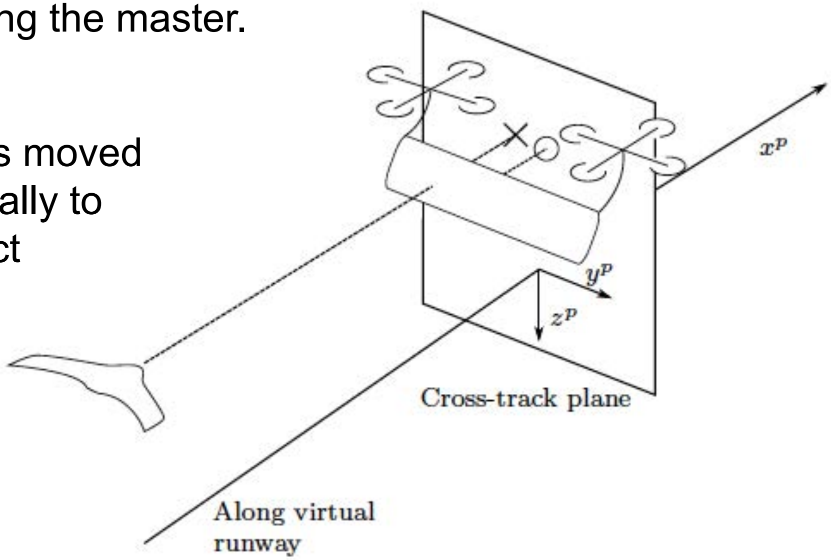
Net Landing onboard a Research Vessel outside the Azores in 2016

The NTNU AMOS Ship-Landing Concept

Cooperative Control: A suspended net is transported away from the ship to a safe distance by using two or more multicopters, which cooperates. They UAVSs are intelligent and share positions in order to control the tension of the net.

Coordinated Control: Master-Slave Principle:
The fixed-wing aircraft acts like an master, while the cooperative multicopters is the slave following the master.

Virtually-Moving Airfield: The airfield/net is moved in an optimal manner vertically and horizontally to improve landing accuracy and reduce impact speed.



K. Klausen, J. B. Moe, J. C. van den Hoorn, A. Gomola, T. I. Fossen and T. A. Johansen. Coordinated Control Concept for Recovery of a Fixed-Wing UAV on a Ship using a Net Carried by Multirotor UAVs. *Proc. of the 2016 International Conference on Unmanned Aircraft Systems* (ICUAS'16), Arlington, VA, 7-10 June, 2016.

The NTNU AMOS Ship-Landing Concept Two or more Multicopters Catches the UAV

Marine Craft

Ref. Encyclopedia Britannica

Marine craft: ships, high-speed craft, semi-submersibles, floating rigs, submarines, remotely operated and autonomous underwater vehicles, torpedoes and other propelled/powered structures for instance a floating air field.

Vehicles that do not travel on land (ocean and flight vehicles) are usually called craft.

Vessel: "hollow structure made to float upon the water for purposes of transportation and navigation; especially, one that is larger than a rowboat ".

The words vessel, ship and boat are often used interchangeably. In Encyclopedia Britannica, a ship and a boat are distinguished by their size through the following definition:

Ship: "any large floating vessel capable of crossing open waters, as opposed to a boat, which is generally a smaller craft. The term formerly was applied to sailing vessels having three or more masts; in modern times it usually denotes a vessel of more than 500 tons of displacement.

Submarine: "any naval vessel that is capable of propelling itself beneath the water as well as on the water's surface.

Underwater Vehicle: "small vehicle that is capable of propelling itself beneath the water surface as well as on the water's surface. This includes unmanned underwater vehicles (UUV), remotely operated vehicles (ROV) and autonomous underwater vehicles (AUV).

Marine Craft

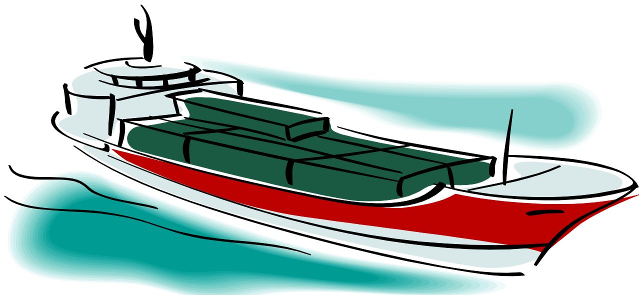
Marine vessels are also classified according to their maximum operating speed. For this purpose it is common to use the Froude number

$$Fn = \frac{U}{\sqrt{gL}}$$

U: ship speed
L: overall (submerged length of the ship)
G: acceleration of gravity

The pressure carrying the vessel can be divided into hydrostatic and hydrodynamic pressure. The corresponding forces are:

- Buoyancy force due to the hydrostatic pressures (proportional to the displacement of the ship)
- Hydrodynamic force due to the hydrodynamic pressure (approximately proportional to the square of the speed)

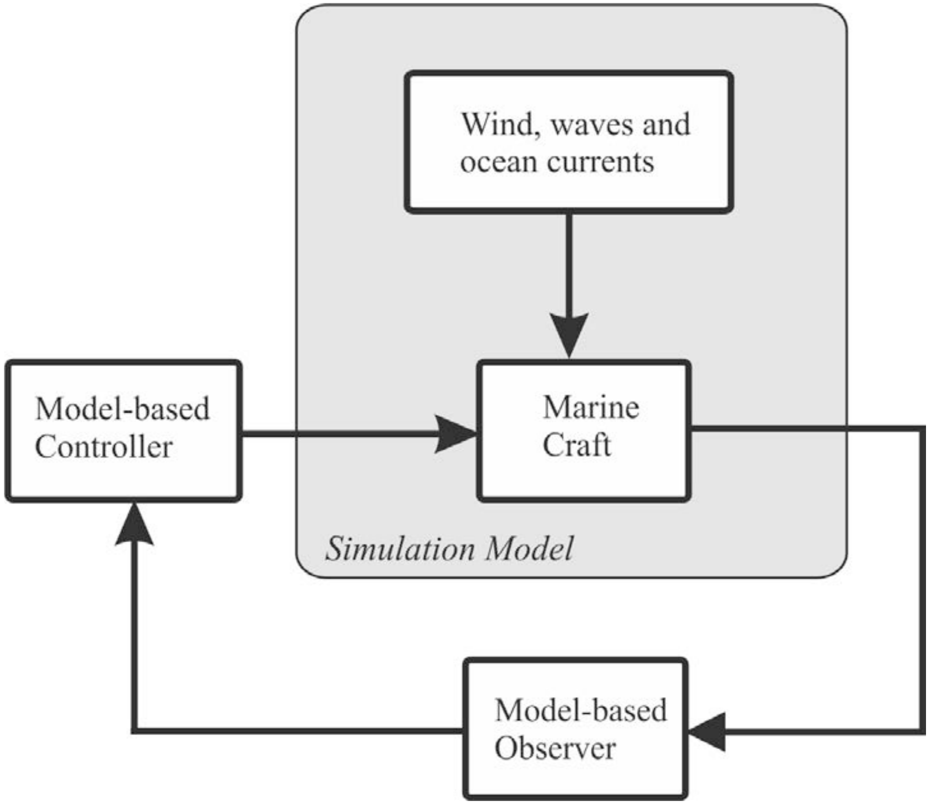


In this course, only displacement vessels are covered

Then we can classify the vessels according to (Faltinsen 2005):

- Displacement vessels ($Fn<0.4$): The buoyancy force dominates .
- Semi-displacement vessel (0.4-$0.5<Fn>1.0$-1.2):
  The buoyancy force is not dominant at the maximum operating speed.
- Planning vessels ($Fn>1.0$-1.2): The hydrodynamic force mainly carries the weight.

Classification of Models



Simulation Model: This model is the most accurate description of a system, for instance a 6-DOF high-fidelity model for simulation of coupled motions in the time domain. It includes the marine craft dynamics, propulsion system, measurement system and the environmental forces due to wind, waves and ocean currents.

The model should be able to reconstruct the time responses of the real system and it should also be possible to trigger failure modes so you can simulate accidents and erroneous signals etc. Simulation models where the fluid memory effects are included (frequency-dependent models) typically consist of 50-200 ODEs while a frequency-independent model can be represented in 6 DOF with 12 ODEs for generalized position and velocity.

In addition you need some states to describe the environmental loads and actuators but still the number of states will be less than 50 for a frequency-independent vessel model

Classification of Models

Control Design Model: The controller model is a reduced-order or simplified simulation model that is used to design the motion control system. In its simplest form, this model is used to compute a set of constant gains for a PID controller. More sophisticated control systems use a dynamic model to generate feedforward and feedback signals. This is referred to as model-based control.

The number of ODEs used in conventional model-based ship control systems is usually less than 20. A PID controller typically requires two states: one for the integrator and one for the low-pass filter used to limit noise amplification. Consequently, setpoint regulation in 6 DOF can be implemented by using 12 ODEs. However, trajectory-tracking controllers require additional states for feedforward as well as filtering so higher-order control laws are not uncommon.

Observer Design Model: The observer model will in general be different from the model used in the controller since the purpose is to capture the additional dynamics associated with the sensors and navigation systems as well as disturbances. It is a simplified version of the simulation model where attention is given to accurate modeling of measurement noise, failure situations including dead-reckoning capabilities, filtering and motion prediction.

For marine craft, the model-based observer often includes a disturbance model where the goal is to estimate wave, wind and ocean current forces by treating these as colored noise. For marine craft the number of ODEs in the state estimator will typically be 20 for a DP system while a basic heading autopilot is implemented with less than 5 states.

The Classical Models in Naval Architecture

The motions of a marine craft exposed to wind, waves and ocean currents are usually modeled in 6 DOF by applying Newton's 2nd law:

$$
\begin{aligned}
m[\dot{u} - vr + wq - x_g(q^2 + r^2) + y_g(pq - \dot{r}) + z_g(pr + \dot{q})] &= X \\
m[\dot{v} - wp + ur - y_g(r^2 + p^2) + z_g(qr - \dot{p}) + x_g(qp + \dot{r})] &= Y \\
m[\dot{w} - uq + vp - z_g(p^2 + q^2) + x_g(rp - \dot{q}) + y_g(rq + \dot{p})] &= Z \\
I_x\dot{p} + (I_z - I_y)qr - (\dot{r} + pq)I_{xz} + (r^2 - q^2)I_{yz} + (pr - \dot{q})I_{xy} & \\
+ m[y_g(\dot{w} - uq + vp) - z_g(\dot{v} - wp + ur)] &= K \\
I_y\dot{q} + (I_x - I_z)rp - (\dot{p} + qr)I_{xy} + (p^2 - r^2)I_{zx} + (qp - \dot{r})I_{yz} & \\
+ m[z_g(\dot{u} - vr + wq) - x_g(\dot{w} - uq + vp)] &= M \\
I_z\dot{r} + (I_y - I_x)pq - (\dot{q} + rp)I_{yz} + (q^2 - p^2)I_{xy} + (rq - \dot{p})I_{zx} & \\
+ m[x_g(\dot{v} - wp + ur) - y_g(\dot{u} - vr + wq)] &= N
\end{aligned}
$$

The Classical Models in Naval Architecture

The external forces and moments X,Y,Z,K,M and N acting on a marine craft are usually modeled by using:

Maneuvering Theory: The study of a ship moving at constant positive speed U in calm water within the framework of maneuvering theory is based on the assumption that the hydrodynamic coefficients are frequency independent (no wave excitation). The maneuvering model will in its simplest representation be linear while nonlinear representations can be derived using methods like cross-flow drag, quadratic damping or Taylor-series expansions.

Seakeeping Theory: The motions of ships at zero or constant speed in waves can be analyzed using seakeeping theory where the hydrodynamic coefficients and wave forces are computed as a function of the wave excitation frequency using the hull geometry. The frequency-dependent models are usually derived within a linear framework while extensions to nonlinear theory is an important field of research.

Maneuvering Theory

Assumes that the ship is moving in restricted calm water. Hence, the maneuvering model is derived for a ship moving at positive speed U under a zero-frequency wave excitation assumption such that added mass and damping can be represented by using hydrodynamic derivatives (constant parameters).

The zero-frequency assumption is only valid for surge, sway and yaw since the natural period of a PD controlled ship will be in the range of 100-150 s. For 150 *s* this result in:

$$\begin{aligned} \omega_n &= \frac{2\pi}{T} \\ &\approx 0.04 \ rad/s \end{aligned}$$

The natural frequencies in heave, roll and pitch are much higher so it is not straightforward to remove the frequency dependence in these channels.

Maneuvering Theory

It is common to formulate the ship maneuvering model as a coupled surge-sway-yaw model and thus neglect heave, roll and pitch motions:

$$
\begin{aligned}
m[\dot{u} - vr - x_g r^2 - y_g \dot{r}] &= X \\
m[\dot{v} + ur - y_g r^2 + x_g \dot{r}] &= Y \\
I_z \dot{r} + m[x_g(\dot{v} + ur) - y_g(\dot{u} - vr)] &= N
\end{aligned}
$$

$$
\mathbf{M}_{RB}\dot{\boldsymbol{\nu}} + \mathbf{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau}_{RB}
$$

$$
\boldsymbol{\tau}_{RB} = \underbrace{\boldsymbol{\tau}_{\text{hyd}} + \boldsymbol{\tau}_{\text{hs}}}_{\text{hydrodynamic and hydrostatic forces}} + \underbrace{\boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}}}_{\text{environmental forces}} + \boldsymbol{\tau}_{\text{control}}
$$

$$
\boldsymbol{\tau}_i = [X_i, Y_i, Z_i, K_i, M_i, N_i]^\top, \qquad i \in \{\text{hyd, hs, wind, wave, control}\}
$$

- Hydrodynamic added mass potential damping due to wave radiation and viscous damping
- Hydrostatic forces (spring stiffness)
- Wind forces
- Wave forces (1st- and 2nd-order)
- Control and propulsion forces

Linearized Maneuvering Models

In the linear 6-DOF case there will be a total of 36 mass and 36 damping elements proportional to velocity and acceleration. In addition to this, there will be restoring forces, propulsion forces and environmental loads. If the generalized force $\boldsymbol{\tau}_{hyd}$ is written in component, the linear added mass and damping forces become:

$$
\begin{aligned}
X_1 &= X_u u + X_v v + X_w w + X_p p + X_q q + X_r r \\
&\quad + X_{\dot{u}} \dot{u} + X_{\dot{v}} \dot{v} + X_{\dot{w}} \dot{w} + X_{\dot{p}} \dot{p} + X_{\dot{q}} \dot{q} + X_{\dot{r}} \dot{r} \\
&\ \vdots \\
N_1 &= N_u u + N_v v + N_w w + N_p p + N_q q + N_r r \\
&\quad + N_{\dot{u}} \dot{u} + N_{\dot{v}} \dot{v} + N_{\dot{w}} \dot{w} + N_{\dot{p}} \dot{p} + N_{\dot{q}} \dot{q} + N_{\dot{r}} \dot{r}
\end{aligned}
$$

where $X_u, X_v, \ldots, N_r$ are the linear damping coefficients and $X_{\dot{u}}, X_{\dot{v}}, \ldots, N_{\dot{r}}$ represent hydrodynamic added mass.

Nonlinear Maneuvering Models

Application of nonlinear theory implies that many elements must be included in addition to the 36 linear elements.

Truncated Taylor-series expansions using odd terms (1st- and 3rd-order) which are fitted to experimental data (Abkowitch 1964) or 2nd-order modulus terms (Fedyaevsky 1963)

$$
\begin{aligned}
X_1 &= X_{\dot{u}}\dot{u} + X_u u + X_{uuu}u^3 + X_{\dot{v}}\dot{v} + X_v v + X_{vvv}v^3 + \cdots \\
&\vdots \\
N_1 &= N_{\dot{u}}\dot{u} + N_u u + N_{uuu}u^3 + N_{\dot{v}}\dot{v} + N_v v + N_{vvv}v^3 + \cdots
\end{aligned}
$$

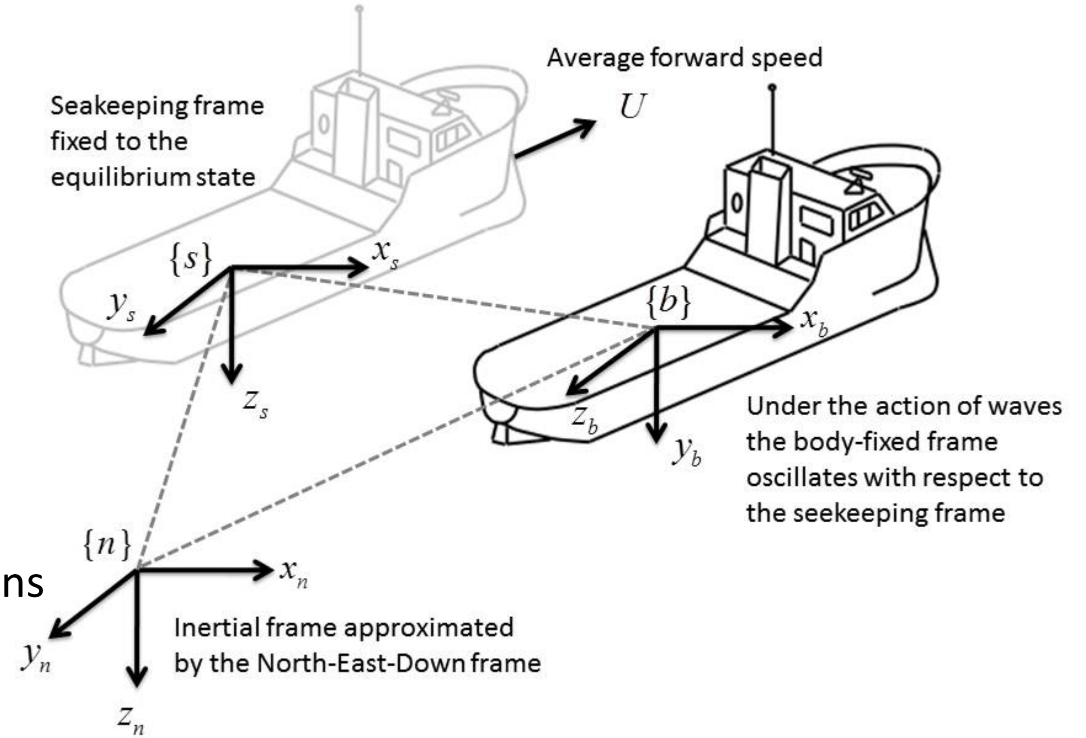
$$
\begin{aligned}
X_1 &= X_{\dot{u}}\dot{u} + X_u u + X_{|u|u}|u|u + X_{\dot{v}}\dot{v} + X_v v + X_{|v|v}|v|v + \cdots \\
&\vdots \\
N_1 &= N_{\dot{u}}\dot{u} + N_u u + N_{|u|u}|u|u + N_{\dot{v}}\dot{v} + N_v v + N_{|v|v}|v|v + \cdots
\end{aligned}
$$

The equations become relatively complicated due to the large number of hydrodynamic coefficients on the right-hand side needed to represent the hydrodynamic forces.

Taylor-series expansions are frequently used in commercial planar motion mechanism (PMM) tests where the purpose is to derive the maneuvering coeffs. experimentally.

Seakeeping Theory

Seakeeping, is the study of motion when there is wave excitation and the craft keeps its course and its speed constant (which includes the case of zero speed). This introduces a dissipative force known as fluid memory effects (Cummins 1962).



The governing model is formulated in the time domain (Cummins equation):

$$[\mathbf{M}_{RB} + \mathbf{A}(\infty)]\ddot{\boldsymbol{\xi}} + \mathbf{B}_{total}(\infty)\dot{\boldsymbol{\xi}} + \int_0^t \mathbf{K}(t-\tau)\dot{\boldsymbol{\xi}}(\tau)d\tau + \mathbf{C}\boldsymbol{\xi} = \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} + \delta\boldsymbol{\tau}$$

Fossen's Robot-Like Vectorial Model for Marine Craft

In Fossen (1991) the robot model:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\mathbf{q} = \boldsymbol{\tau}$$

- **q** is a vector of joint angels
- $\boldsymbol{\tau}$ is a vector of torque
- **M** and **C** are the system inertia and Coriolis matrices

was used as foundation to write the 6-DOF marine craft equations of motion in a compact vectorial setting.

Matrix-Vector Representation

The robot model was modified to describe marine craft according to:

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{g}(\boldsymbol{\eta}) + \mathbf{g}_0 = \boldsymbol{\tau} + \boldsymbol{\tau}_{wind} + \boldsymbol{\tau}_{wave}$$

- body-fixed velocities: $\boldsymbol{\nu} = [u, v, w, p, q, r]^T$
- position and Euler angles: $\boldsymbol{\eta} = [x, y, z, \phi, \theta, \psi]^T$
- **M, C** and **D** denote the system inertia, Coriolis and damping matrices
- **g** is a vector of gravitational and buoyancy forces and moments

Why use Matrix-Vector Form?

It is advantageous to express the equations of motion in matrix-vector form:

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{g}(\boldsymbol{\eta}) + \mathbf{g}_0 = \boldsymbol{\tau} + \boldsymbol{\tau}_{wind} + \boldsymbol{\tau}_{wave}$$

since nonlinear system properties such as:

- symmetry of matrices
- skew-symmetry of matrices
- positiveness of matrices

can be exploited in the passivity/stability analysis. These properties relates to passivity of the model. Notice thThese properties are destroyed by linearization.

These properties also represents physical properties of the system which should be exploited when designing nonlinear controllers and observers for marine craft.

The matrix-vector notation makes it very easy to simulate the model (in Matlab and C++)

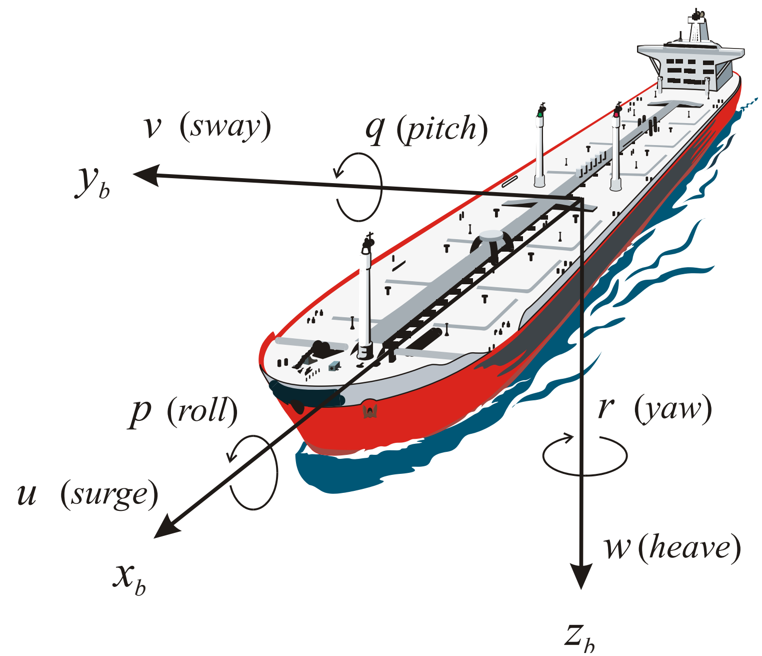
Degrees of Freedom (DOF)

DOF		forces and moments	linear and angular velocities	positions and Euler angles
1	motions in the x-direction (surge)	X	u	x
2	motions in the y-direction (sway)	Y	v	y
3	motions in the z-direction (heave)	Z	w	z
4	rotation about the x-axis (roll, heel)	K	p	ϕ
5	rotation about the y-axis (pitch, trim)	M	q	θ
6	rotation about the z-axis (yaw)	N	r	ψ

The notation is adopted from ***SNAME (1950).***

For a marine craft, DOF is the set of independent displacements and rotations that completely specify the displaced position and orientation of the craft. A craft that can move freely in the 3-D space has maximum 6 DOFs—three translational and three rotational components.

Consequently, a fully actuated marine craft operating in 6 DOF must be equipped with actuators that can produce independent forces and moments in all directions.



Degrees of Freedom (DOF)

When designing feedback control systems for marine craft, reduced-order models are often used since most vehicles do not have actuation in all DOF. This is usually done by decoupling the motions of the vessel according to:

1-DOF models can be used to design forward speed controllers (surge), heading autopilots (yaw) and roll damping systems (roll).

3-DOF models are usually horizontal-plane models (surge, sway and yaw) for ships, semi-submersibles and underwater vehicles that are used in DP systems, trajectory-tracking control systems and path-following systems. For slender bodies such as torpedo-shaped AUVs and submarines, it is also common to assume that the motions can be decoupled into longitudinal and lateral motions.

- Longitudinal models (surge, heave and pitch) for forward speed, diving and pitch control.
- Lateral model (sway, roll and yaw) for turning and heading control.

4-DOF models (surge, sway, roll and yaw) are usually formed by adding the roll equation to the 3-DOF horizontal-plane model. These models are used in maneuvering situations where the purpose is to reduce roll by active control of fins, rudders or stabilizing liquid tanks.

6-DOF models (surge, sway, heave, roll, pitch and yaw) are fully coupled equations of motion used for simulation and prediction of coupled vessel motions. These models can also be used in advanced control systems for underwater vehicles, which are actuated in all DOF.